Theoretical modelling of multiple OWCs along a straight coast/breakwater

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1. Introduction

Integration of the OWC with other marine structures, such as a breakwater, seawall or jetty, presents an effective way to increase the attractiveness of wave power exploitation. In such a way, the economics of the OWC can be enhanced due to the cost sharing benefits including construction, installation and maintenance. Reliability and survivability of the OWC can be improved as well. Many theoretical investigations have been devoted to wave power extraction by coast/breakwater integrated OWCs (Martins-rivas and Mei, 2009a, 2009b; Lovas et al., 2010). To fully harness the available wave power and to produce large quantities of electricity for electrical grids, some researchers believe that an array of OWCs is likely to be deployed (Nihous, 2012; Konispoliatis and Mavrakos, 2016).

In this article, a concept by integrating multiple OWCs with a straight coast/breakwater is proposed. The chamber of each OWC mainly consists of a vertical circular cylinder with a ring shape cross section, which is half open to the sea from a finite submergence to the seabed. To evaluate hydrodynamic performance of these coast/breakwater integrated OWCs, a theoretical model is developed based on the linear potential flow theory and eigen-function matching method. Effect of the Well turbine installed at the top of each OWC together with the air compressibility are taken into account as a linear power take-off (PTO) system. The influence of incident wave direction, chamber size, spacing between the OWCs and number of OWCs on power extraction are systematically investigated by applying the present theoretical model.

2. Mathematical model

Consider $N$ vertical circular OWCs installed along a straight coast/breakwater in water of finite depth $h$ (see Fig.1, $N=2$ is taken as an example), $N$ local cylindrical coordinates are defined with their origins $O_n$, located on the central vertical axis of the $n$-th OWC. The out radius, inner radius and draft of the $n$-th OWC chamber are denoted by $R_n$, $R_{in}$ and $d_n$, respectively. $\beta$ denotes the incident direction.

![Fig. 1. Definition sketch: (a) bird view; (b) top view.](image)

Within the framework of linear potential flow theory, the fluid flow in the water domain can be described by the velocity potential, and the spatial velocity potential $\Phi$ can be decomposed as

$$\Phi = \Phi_1 + \Phi_0 + \sum_{n=1}^{N} p_n \Phi_n,$$  \hspace{1cm} (1)

where $\Phi_1$ is the wave spatial potential when the vertical wall with no OWCs subjected to incident waves; $\Phi_0$ denotes the diffracted wave spatial potential disturbed by the existence of OWCs; $p_n$ is the complex air pressure amplitude inside the $n$-th OWC chamber; and $\Phi_n$ represents the spatial velocity potential due to unit air pressure oscillation inside the $n$-th...
OWC chamber. Hereinafter, the so-called scattering velocity potential \( \Phi_0 \) is adopted to represent the sum of \( \Phi_1 \) and \( \Phi_0 \), i.e., \( \Phi_0=\Phi_1+\Phi_2 \).

In different subdomains, the velocity potentials \( \Phi_x (x=0, 1, \ldots, N) \) can be expressed as follows:

(1) Inner domain, the region enclosed by the \( n \)-th OWC

\[
\Phi_{x,n}^i(r, \theta, z) = \sum_{m=-\infty}^{n} \sum_{l=0}^{n} K_m(klr) A_{m,l,j}^n \cos[m\theta] Z_j(z) e^{i\phi} - \frac{i\delta_{m,0}^n}{\rho \omega},
\]

(2) Semi-ring domain, the region beneath the \( n \)-th OWC chamber

\[
\Phi_{x,n}^i(r, \theta, z) = \delta_{x,0}^n \Phi + \sum_{m=0}^{n} \sum_{l=0}^{n} E_{m,l,j}^n \tilde{K}_m(klr) \cos(m\theta) Z_j(z) \cos(m'\theta) + \sum_{j=1}^{n} \sum_{m=-\infty}^{n} \sum_{l=0}^{n} \tilde{K}_m(klr) \tilde{I}_m(klr) e^{i(mz+nm\omega)} \cos(m'\theta)
\]

where

\[
F_{n,0}^i(r) = \left\{ \begin{array}{ll}
C_{m,0,j}^n + D_{m,0,j}^n \left[ 1 + \ln \left( \frac{r}{R} \right) \right] & , \quad m = 0 \\
C_{m,0,j}^n \left( \frac{r}{R} \right)^{m+1} + D_{m,0,j}^n \left( \frac{r}{R} \right)^{m+1} & , \quad m \neq 0
\end{array} \right.
\]

(5)

\[
I_m(klr) = \left\{ \begin{array}{ll}
J_m(klr) & , \quad l = 0 \\
I_m(klr) & , \quad l \neq 0
\end{array} \right.
\]

\[
K_m(klr) = \left\{ \begin{array}{ll}
H_m(klr) & , \quad l = 0 \\
K_m(klr) & , \quad l \neq 0
\end{array} \right.
\]

\[
Z_j(z) = N_{0,j} c \cos[k_0(z+h)] , \quad N_{0,j} = \frac{1}{2} \left[ 1 + \frac{\sin(2kh)}{2kh} \right],
\]

(7)

\[
Z_j(z) = N_{1,j} \sin[k_0(z+h)] , \quad N_{1,j} = \frac{1}{2} \left[ 1 - \frac{\sin(2kh)}{2kh} \right],
\]

(8)

\( A_{m,l,j}^n, C_{m,l,j}^n, D_{m,l,j}^n \) and \( E_{m,l,j}^n \) are the unknown coefficients to be solved; \( k \) and \( \beta_n \) are the eigenvalues, where \( k_0=k \) is the wave number; \( J_m, I_m, H_m \) and \( K_m \) denote the Bessel function, the modified Bessel function of the first kind, the Hankel function of the first kind and the modified Hankel function of the second kind, respectively; \( R_m \) and \( a_m \) are the norm and the angle of vector \( \text{O}_m\text{O}_n \), respectively. The pressure and velocity continuity conditions on the interfaces of each two adjacent regions are used to solve the unknown coefficients. The wave excitation volume flux of the \( n \)-th OWC, denoted as \( Q_n \), the volume flux of the \( n \)-th OWC due to the radiated velocity potential induced by the unit air pressure oscillation inside the \( x \)-th chamber, denoted as \( Q_n = -\left( \epsilon_n^{(n)} - i\eta_n^{(n)} \right) \), where \( \epsilon_n^{(n)} \) and \( \eta_n^{(n)} \) are the so-called hydrodynamic coefficients, can be calculated straightforward.

PTO mass, \( \alpha_{\text{PTO}} = \omega V_0/(
u^2 \rho_0) \), is calculated based on \( \rho_\omega \rho_0 = 1000, \nu = 340 \text{ m/s}, h = 10 \text{ m}, \) and \( V_0 = \pi R^2 h \), where \( V_0, \nu \) and \( \rho_0 \) are air volume in the chamber, sound velocity in air and static air density, respectively. The corresponding optimal PTO damping, \( \alpha_{\text{PTO}} \), is considered equal to the optimum coefficient of the same coast/breakwater integrated OWC when working in isolation condition (Lovas et al., 2010). The dimensionless coefficients are defined as:

\[
\tilde{Q}_n = \sqrt{\frac{g}{h}} \sqrt{\frac{\rho \omega}{h \Omega} \left( \frac{A_{\text{Hg}}}{n} \right)} \left( \tilde{\epsilon}_n, \tilde{\eta}_n \right) = \left( \epsilon_n^{(n)}, \eta_n^{(n)} \right) \left( \epsilon_n^{(n)}, \eta_n^{(n)} \right) \rho \sqrt{\frac{g}{h}},
\]

(9)

and the time-averaged power absorption of the \( N \) OWCS can be written in terms of wave capture factor \( \eta_0 \), which is defined as \( k \) times of the wave capture width. \( q \)-factor is adopted as well to evaluate effect of the hydrodynamic interaction among the OWCS on power extraction: \( q = \eta/(N \eta_0) \), where \( \eta_0 \) represents the maximum wave capture factor of an individually isolated coast/breakwater integrated OWC.
3. Results and discussions

The excellent agreement of wave damping coefficients of two coast/breakwater integrated OWCs with the same size and the distance $D=O_1 O_2=2.0h$ by using direct method and the indirect method based on Haskind Relation, as given in Fig.2, proves the correctness of the present theoretical model.

![Fig. 2. Results of wave damping coefficients by using direct method and the indirect method based on Haskind Relation, $R/h=0.5$, $(R-R)/h=0.1$, $d/h=0.2$, $D/h=2.0$. (a) $\tilde{c}_1^{(1)}$; (b) $\tilde{c}_2^{(1)}$.](image)

The influences of incident wave direction, chamber size, spacing between the OWCs and the number of OWCs on power extraction are all explored by using the validated theoretical model. Here, we focus mainly on the comparison and discussion of the results for the case of two coast/breakwater integrated OWCs deployed at different spacings $D/h$, and $R/h=0.5$, $(R-R)/h=0.1$, $d/h=0.2$, $\beta=\pi/2$, as given in Fig. 3. Other results will be presented in the workshop. Results of the individually isolated coast/breakwater integrated OWC (denoted as “isolated”) are also displayed in Fig. 3 as a comparison.

Fig.3a shows that there are two peaks of $|Q_r^{(\eta)}|kh$ curve over the computed range of $kh$, with the main one at $kh=1.8$ and the second sharp one at a higher frequency, i.e., $kh=4.82$. As $D/h$ increases from 1.5 to 3.0, the amplitude of the main peak first increases and then decreases. Although the amplitude of the main peak for $D/h=1.5$ is merely 2.4, large values of $|Q_r^{(\eta)}|$ are obtained at $kh=(2.1, 3.0)$. The $kh$ corresponding to the main peak shifts toward lower frequencies. The second sharp peak is nearly independent of $D/h$.

As displayed in Figs. 3b and 3d, a rather limited impact of $D/h$ on $\tilde{c}_1^{(1)}$ and $\tilde{a}_1^{(1)}$ is observed at $kh=(1.5, 2.5)$, where the main peak of $\tilde{c}_1^{(1)}$-$kh$ curve and the corresponding drop of $\tilde{a}_1^{(1)}$ occur. As $D/h$ varies, the $\tilde{c}_1^{(1)}(\tilde{a}_1^{(1)})$-$kh$ curve of the two OWCs oscillates slightly around that of the “isolated” case. This is due to the fact that the waves radiated from each coast/breakwater integrated OWC and also those diffracted from the other OWC act on the OWC simultaneously. The change of $D/h$ leads to alteration of the phase difference between the two-OWC mutual radiated and diffracted waves, resulting in the switch of reinforcing and diminishing influences. The amplitudes of the peak of $\tilde{c}_1^{(1)}$ and the drop of $\tilde{a}_1^{(1)}$ at $kh=1.8$ are both approx. 3.5. As a comparison (Figs. 3c and 3e), the variations of $\tilde{c}_2^{(1)}$ and $\tilde{a}_2^{(1)}$, especially for $kh=(1.0, 3.0)$, are significantly dependent upon $D/h$. The amplitudes of the drops of $\tilde{c}_2^{(1)}$ and $\tilde{a}_2^{(1)}$ around $kh=1.8$ are found to be both no smaller than 1.6, revealing a strong hydrodynamic interaction between the OWCs for the four cases of $D/h$ examined. As $D/h$ increases from 1.5 to 3.0, these drops of $\tilde{c}_2^{(1)}$ and $\tilde{a}_2^{(1)}$ turn progressively weaker and weaker, and it can be expected that for $D/h\rightarrow\infty$, $\tilde{c}_2^{(1)}=0$ and $\tilde{a}_2^{(1)}=0$ will be obtained.

Figs. 3f and 3g present the frequency response of the wave capture factor ($\eta$), and $q$-factor, respectively. $\eta$ for the “isolated” case, i.e., $\eta_0$, is no more than 2.0 (see Fig. 3f). For the cases consisting of two OWCs, thanks to the hydrodynamic interaction, $\eta>6.0$ can be obtained for some specified values of $D/h$. From the point of view of the peak value of $\eta$, the OWCs with $D/h=2.0$ could the best in power absorption. However, in practice, the OWCs with $D/h=1.5$ might be a better choice for their good performance in a broader bandwidth, with a rather large wave capture factor. It can be learnt from Fig. 3g that, for $D/h=1.5$, $q>1.0$ is satisfied at $kh=(1.8, 3.4)$, meaning that a constructive hydrodynamic interaction between the OWCs is achieved in such a large range of wave conditions.
Fig. 3. Comparison for different spacings between the OWCs, $D/h$, with $R/h=0.5$, $(R-R_i)/h=0.1$, $d/h=0.2$, $\beta=\pi/2$. (a) $\bar{Q}_e^\beta$; (b) $\bar{r}_1^\beta$; (c) $\bar{r}_2^\beta$; (d) $\bar{a}_\psi^\beta$; (e) $\bar{\pi}_1^\beta$; (f) $\eta$; (g) $q$.

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References


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