Statistical characterisation of wind fields over complex terrain for bushfire modelling

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Motivation

• With emerging ensemble-based fire risk modelling frameworks, it is useful to recast wind in probabilistic terms.
• Probabilistic fire modelling inputs allow for better informed decision making when uncertainties are quantified and accounted for.

Source: French et al. (2013)
Directional Wind Response

Prevailing Wind Direction

Joint Directional Wind Response

Wind Direction on West Slope

Prevailing Wind Direction

Wind Direction on East Slope

Joint Directional Wind Response
Flea Creek Valley

January to October 2007 and April to December 2014

Point 1
Point 2
Point 3
Point 4

WNW

2007 Data: Sharples et al. (2010)
Statistical Comparison Tests

Consider the empirical distributions

– Statistics are based on the maximum difference between the cumulative distributions.

– Further work will consider the adaptation of this statistic to account for circularity.
Kolmogorov-Smirnov Test

• Univariate – maximum difference between the empirical distributions

\[ D_{n}^{(1)} = \sup_{x} |F_{X}(x) - G_{X}(x)|, \text{ where } F_{X}(x) = P(X \leq x) \]

• Since this is proportional to \( n \), an the following alternative is used

\[ Z_{n}^{(1)} = \sqrt{n}D_{n}^{(1)}, \text{ with } n = \frac{n_{1}n_{2}}{n_{1} + n_{2}} \]

• Critical Values of \( D_{n}^{(1)} \) (Massey, 1951)

\[ d_{0.01} = 1.63 / \sqrt{n}, \quad d_{0.05} = 1.36 / \sqrt{n} \]

• P-values
  – (Gosset, 1987)

\[ P(Z_{\infty}^{(1)} > z) \approx 2 \exp(-2z^{2}) \]
  – Monte Carlo simulations (\( M = 1000 \)
Kolmogorov-Smirnov Test

<table>
<thead>
<tr>
<th></th>
<th>$n_1$</th>
<th>$n_2$</th>
<th>$D_{n}^{(1)}$</th>
<th>$d_{0.01}$</th>
<th>$d_{0.05}$</th>
<th>$Z_{n}^{(1)}$</th>
<th>$P_Z$</th>
<th>$P_{Z_{+}}^{\infty}$</th>
<th>$P_{D_{n}}^{\infty}$</th>
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</thead>
<tbody>
<tr>
<td>Point 1</td>
<td>1046</td>
<td>403</td>
<td>0.2259</td>
<td>0.0956</td>
<td>0.0797</td>
<td>3.8529</td>
<td>2.55 E-33</td>
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<td>0</td>
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<tr>
<td>Point 2</td>
<td>129</td>
<td>399</td>
<td>0.1630</td>
<td>0.1651</td>
<td>0.1377</td>
<td>1.6096</td>
<td>0.0112</td>
<td>0.009</td>
<td>0.001</td>
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<tr>
<td>Point 3</td>
<td>825</td>
<td>411</td>
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<td>0.0821</td>
<td>6.9987</td>
<td>5.7 E-43</td>
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</tr>
<tr>
<td>Point 4</td>
<td>903</td>
<td>338</td>
<td>0.4893</td>
<td>0.1057</td>
<td>0.0882</td>
<td>7.6740</td>
<td>1.41 E-51</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

Point 1: Leeward Slope  
Point 2: Valley Floor  
Point 3: Windward Slope  
Point 4: Windward Slope

Surface Wind Direction, Conditional on WNW Prevailing Wind Direction

2007

2014
Extended Kolmogorov-Smirnov Test

• With a bivariate joint distribution, we can define the CDF in four directions (Peacock, 1983):

\[ Q_1 = (X \leq x, Y \leq y), \quad Q_2 = (X \leq x, Y \geq y), \quad Q_3 = (X \geq x, Y \leq y), \quad Q_4 = (X \geq x, Y \geq y) \]

• So the bivariate extension of the KS statistic becomes the maximum of the maximum differences between empirical distributions

\[
D_n^{(2)} = \max (D_n^{Q_1}, D_n^{Q_2}, D_n^{Q_3}, D_n^{Q_4})
\]

with

\[
D_n^{Q_1} = \sup_{x,y} |F_{X,Y}^{Q_1}(x,y) - G_{X,Y}^{Q_1}(x,y)|, \quad \text{where} \quad F_{X,Y}^{Q_1}(x,y) = P(X \leq x, Y \leq y)
\]

• This is still proportional to \(n\), so the following alternative is used

\[
Z_n^{(2)} = \sqrt{n} D_n^{(2)}, \quad \text{with} \quad n = \frac{n_1 n_2}{n_1 + n_2}
\]
Extended Kolmogorov-Smirnov Test

• P-values
  – For the area of interest where \( P (Z_n^{(2)} > z) \leq 0.2 \)
    the asymptotic behaviour of the statistic is given by (Peacock, 1983);
    \[
    P (Z_{\infty}^{(2)} > z) \approx 2 \exp (-2(z - 0.5)^2)
    \]
  – Monte Carlo simulations?

• Critical Values?
  – Peacock (1983) gives critical values for \( D_n^{(2)} \) with \( n = 50 \);
    \[
    d_{0.01} = 2.06, \quad d_{0.05} = 1.83
    \]
  – But we have much larger sample sizes...
Extended Kolmogorov-Smirnov Test

<table>
<thead>
<tr>
<th>Point</th>
<th>( n_1 )</th>
<th>( n_2 )</th>
<th>( D_{n}^{(1)} )</th>
<th>( Z_{n}^{(1)} )</th>
<th>( P_Z )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Point 1</td>
<td>2537</td>
<td>2809</td>
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<td>12.0804</td>
<td>6.58 E-117</td>
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<tr>
<td>Point 2</td>
<td>346</td>
<td>2823</td>
<td>0.2931</td>
<td>5.1466</td>
<td>3.53 E-19</td>
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<tr>
<td>Point 3</td>
<td>1676</td>
<td>2964</td>
<td>0.4574</td>
<td>14.9673</td>
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<td>Point 4</td>
<td>1864</td>
<td>2161</td>
<td>0.4617</td>
<td>14.6070</td>
<td>2.79 E-173</td>
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</table>

For \( n = 50 \),
\[ d_{0.01} = 2.06, \]
\[ d_{0.05} = 1.83. \]

**Point 1: Leeward Slope**

**Point 2: Valley Floor**

**Point 3: Windward Slope**

**Point 4: Windward Slope**

Discrete Observed Joint Wind Direction Distributions
Kuiper’s Test

• Accounts for circularity (Kuiper, 1960)

\[ V_n^{(1)} = \sup_x \{ F_X(x) - G_X(x) \} - \inf_x \{ F_X(x) - G_X(x) \} \]

• Extension to Bivariate as in KS?

\[ V_n^{(2)} = \max_i (V_n^{Q_i}) \text{, or} \]
\[ V_n^{(2)} = \max_i (V_n^{Q_i}) - \min_i (V_n^{Q_i}) \] ?

• P-values and critical values...
**HOW** has the vegetation altered the wind fields across Flea Creek Valley?

(1) Evaluate the sensitivity of the tests using simulation studies

(2) Consider a more controlled experiment
Sensitivity Evaluation

How big does a change in the distribution need to be to cause a significant test result?
Sensitivity Evaluation

Initial univariate, uni-modal results for Normal distribution

<table>
<thead>
<tr>
<th>Model 1</th>
<th>Model 2</th>
<th>$n_1$</th>
<th>$n_2$</th>
<th>$D_{n}^{(1)}$</th>
<th>$d_{0.01}$</th>
<th>$d_{0.05}$</th>
<th>$Z_{n}^{(1)}$</th>
<th>$P_{Z}$</th>
<th>$P_{Z}^{in}$</th>
<th>$P_{D}^{in}$</th>
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<tr>
<td>N(8,1)</td>
<td>N(7,1)</td>
<td>706</td>
<td>837</td>
<td>0.3713</td>
<td>0.0833</td>
<td>0.0695</td>
<td>7.2669</td>
<td>2.71 E-46</td>
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<td>N(7.5,1)</td>
<td>N(7,1)</td>
<td>277</td>
<td>604</td>
<td>0.1973</td>
<td>0.1183</td>
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<td>2.7196</td>
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<tr>
<td>N(8,1)</td>
<td>N(8,1)</td>
<td>678</td>
<td>485</td>
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<td>0.0969</td>
<td>0.0809</td>
<td>0.5780</td>
<td>0.8920</td>
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<td>0.3450</td>
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<tr>
<td>N(8,1)</td>
<td>N(8.5,1)</td>
<td>852</td>
<td>561</td>
<td>0.1976</td>
<td>0.0886</td>
<td>0.0739</td>
<td>3.6342</td>
<td>6.75 E-12</td>
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</tr>
<tr>
<td>N(9,1)</td>
<td>N(8,1)</td>
<td>624</td>
<td>1048</td>
<td>0.3978</td>
<td>0.0824</td>
<td>0.0688</td>
<td>7.8680</td>
<td>3.4 E-54</td>
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<tr>
<td>N(8,1)</td>
<td>N(8,0.9)</td>
<td>968</td>
<td>905</td>
<td>0.0261</td>
<td>0.0754</td>
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<td>0.5639</td>
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<tr>
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<td>N(8,0.8)</td>
<td>755</td>
<td>1007</td>
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<td>0.0785</td>
<td>0.0655</td>
<td>1.2752</td>
<td>0.0778</td>
<td>0.0080</td>
<td>0.0080</td>
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<tr>
<td>N(8,0.75)</td>
<td>N(8,0.75)</td>
<td>458</td>
<td>917</td>
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<td>0.0933</td>
<td>0.0778</td>
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<td>0.0015</td>
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<tr>
<td>N(8,0.5)</td>
<td>N(8,0.5)</td>
<td>640</td>
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<td>0.0693</td>
<td>4.0054</td>
<td>2.32 E-14</td>
<td>0</td>
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</tr>
</tbody>
</table>

Histograms of Sample 1, Sample 2, and Sample 2 with different models.
Sensitivity Evaluation

Continuing Work

• Univariate distributions
  – Bi-modal
  – Circular

• Bivariate distributions
  – Bivariate Normals,
  – Wrapped Normals or von Mises
  – Mixtures for multimodal distributions
Controlled Study: National Arboretum Canberra

April 2015 to Present
NAC: Changes in Vegetation

Wind Direction on Ridge Top

Wind Direction on Clear Slope

Wind Direction on Pine Slope

PAWS6

PAWS7

PAWS9

PAWS3
NAC: Changes in Topography

![Image of topographic changes with markers and graphs showing wind direction on different slopes.](image-url)
Further Work

– Continue and extend investigations to allow better physical interpretation of results in relation to wind fields.

– Consideration of the impacts of vegetation on wind speeds, not just wind directions.

– Evaluate current operational models using observed data.

Consider the potential for hybrid probabilistic approach to wind modelling for bushfire applications.
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Thank you

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References