CARMA and Me for 18-08-2012
CARMA Annual Retreat

Jonathan M. Borwein FRSC FAA FAAAS

Laureate Professor & Director of CARMA, University of Newcastle

Priority Research Centre for
Computer Assisted Research Mathematics and its Applications

Revised: August 17 2012
Please:

1. Bookmark this Home page
2. Regularly monitor Events
   - and make sure they are advertised
3. Report Issues to
   - David Allingham and Roslyn Hickson
4. Post News Items
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Contents. We will *sample* the following:

1. 3. CARMA’s Mandate
   3. Experimental Mathematics
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   9. CARMA’s Objectives
   10. Communication, Computation and Collaboration

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   11. CARMA’s Background
   12. CARMA Structure
   13. CARMA Activities
   14. CARMA Services

3. 15. My Current Research
   15. My Current Interests: **SNAG** and the like
   17. Some Mathematics and Related Images
   19. A Short Ramble: Density of short random walks
   26. Why Pi? Frivolity, utility and normality
   30. Pi seems Random: walking on numbers

4. 37. Modern Mathematical Visualization
   37. Animation, Simulation and Stereo
   38. Conclusion
Experimental mathematics is the use of a computer to run computations—sometimes no more than trial-and-error tests—to look for patterns, to identify particular numbers and sequences, to gather evidence in support of specific mathematical assertions that may themselves arise by computational means, including search.

Like contemporary chemists — and before them the alchemists of old—who mix various substances together in a crucible and heat them to a high temperature to see what happens, today’s experimental mathematicians put a hopefully potent mix of numbers, formulas, and algorithms into a computer in the hope that something of interest emerges. (JMB-Devlin, 2008, p. 1)

• Quoted in International Council on Mathematical Instruction Study 19: On Proof and Proving, 2012
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J.M. Borwein  CARMA and Me, 2012
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$$a_0 \beta + a_1 \alpha_1 + a_2 \alpha_2 + \cdots + a_n \alpha_n = 0,$$

where $a_i$ are integers—if one exists and provides an exclusion bound otherwise.

- If $a_0 \neq 0$ then (1) assures $\beta$ is in rational vector space generated by $\{\alpha_1, \alpha_2, \ldots, \alpha_n\}$.
- $\beta = 1, \alpha_i = \alpha^i$ means $\alpha$ is algebraic of degree $n$.
- 2000 *Computing in Science & Engineering*: PSLQ one of top 10 algorithms of 20th century

**Madelung constant**

**CMS D.Borwein Prize**

**Profile: Helaman Ferguson**

Carving His Own Unique Niche, In Symbols and Stone

By refusing to choose between mathematics and art, a self-described “mistfit” has found the place where parallel careers meet.
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Top Ten Algorithms: all but one well used in CARMA

Algorithms for the Ages

"Great algorithms are the poetry of computation," says Francis Sullivan of the Institute for Defense Analyses' Center for Computing Sciences in Bowie, Maryland. He and Jack Dongarra of the University of Tennessee and Oak Ridge National Laboratory have put together a sampling that might have made Robert Frost beam with pride—had the poet been a computer jock. Their list of 10 algorithms having "the greatest influence on the development and practice of science and engineering in the 20th century" appears in the January/February issue of Computing in Science & Engineering. If you use a computer, some of these algorithms are no doubt crunching your data as you read this. The drum roll, please:

1. **1946: The Metropolis Algorithm for Monte Carlo.** Through the use of random processes, this algorithm offers an efficient way to stumble toward answers to problems that are too complicated to solve exactly.
2. **1947: Simplex Method for Linear Programming.** An elegant solution to a common problem in planning and decision-making.
3. **1950: Krylov Subspace Iteration Method.** A technique for rapidly solving the linear equations that abound in scientific computation.
5. **1957: The Fortran Optimizing Compiler.** Turns high-level code into efficient computer-readable code.
7. **1962: Quicksort Algorithms for Sorting.** For the efficient handling of large databases.
8. **1965: Fast Fourier Transform.** Perhaps the most ubiquitous algorithm in use today, it breaks down waveforms (like sound) into periodic components.
10. **1987: Fast Multipole Method.** A breakthrough in dealing with the complexity of n-body calculations, applied in problems ranging from celestial mechanics to protein folding.

Experimental Mathematics: PSLQ is core to CARMA

Exploratory Experimentation and Computation

David H. Bailey and Jonathan M. Borwein

The authors’ thesis—once controversial, but now a commonplace—is that computers can be a useful, even essential, aid to mathematical research.

Jeff Shallit wrote this in his recent review (96/214:760:3) of [111]. As we hope to make clear, Shallit was entirely right in that many, if not most, research mathematicians now use the computer in a variety of ways to draw pictures, respect numerical data, manipulate expressions symbolically, and run simulations. However, it seems as if there has not yet been substantial and intellectually rigorous progress in the way mathematics is presented in research papers, works, and classroom instruction or as the mathematical discovery process is organized.

Mathematicians Are Humans

We share with George Polya (1887–1985) the view that, while learning, we must come to see that there are some methods with much less outside influence than formal arguments.

Polya went on to reframe, nonetheless, that proof should certainly be sought in school. We return to observations, many of which have been found in published books such as Mathematics by Experiment [10] and Experimental Mathematics in Action [11], in which we have noted the changing nature of mathematical knowledge and in consequence ask questions such as “How do we teach what and why to students?”, “How do we come to believe and trust these pieces of mathematics?”, and “Why do we do things?” An answer to the last question is “That depends.” Sometimes we wish insight and sometimes, with subsidiary results, we are more than happy with a certificate. The computer has significant capabilites to assist with both.

Nestor (27, p. 113) defines:

the large human brain evolved over the past 7 million years to allow individuals to separate the growing complexity found by human social living.

As a result, humans have a variety of modes of argument more powerful than others and are more prone to make certain kinds of errors than others. Likewise, the well-known evolutionary psychologist Steven Pinker observes that language [24, p. 404] is based on

the ethical notions of space, time, causation, perspective, and goals that appear to make up a language of thought.

This remains so within mathematics. The computer offers a framework both to enhance mathematical reasoning, as is the recent computation connected to the Lio group by [http://www.aimath.org/EX/computation/1a.html, to reduce the following result derived in [14]:

1. \[
\sum_{n=1}^{\infty} \frac{(-1)^n}{n^2} = \frac{\pi^2}{12} - \frac{1}{e^2} + \frac{1}{2!} - \frac{1}{3!} + \frac{1}{4!} - \frac{1}{5!} + \cdots
\]

2. \[
\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^3} = \frac{\pi^3}{32} - \frac{1}{24} + \frac{1}{2!} - \frac{1}{3!} + \frac{1}{4!} - \frac{1}{5!} + \cdots
\]

Here \[\text{Li}(x)\] is a polylogarithmic value. However, a subsequent computation to check results indicated that, whereas the LHS evaluates to \[0.879729289\ldots\], the RHS evaluates to \[2.593380819\ldots\]. Puzzled, we computed the sum, as well as each of the terms on the RHS (using their coefficients), to 50-digit precision, then applied the “Pi Interpretive Algorithm,” which searches for integer relations among a set of constants [16]. Perhaps one would have thought that

the following result was well-known:

J.M. Borwein

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J.M. Borwein
CARMA and Me, 2012
CARMA’s Mandate

Mathematics, as “the language of high technology” which underpins all facets of modern life and current Information and Communication Technology (ICT), is ubiquitous. No other research centre exists focussing on the implications of developments in ICT, present and future, for the practice of research mathematics.

- CARMA fills this gap through exploitation and development of techniques and tools for computer-assisted discovery and disciplined data-mining including mathematical visualization.
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CARMA’s Access Grid Room

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CARMA’s Objectives:

To perform R&D relating to the informed use of computers as an adjunct to mathematical discovery (including current advances in cognitive science, in information technology, operations research and theoretical computer science).

- of mathematics underlying computer-based decision support systems, particularly in automation and optimization of scheduling, planning and design activities, and to undertake mathematical modelling of such activities. (C-OPT, NUOR and partners)

- To promote and advise on use of appropriate tools (hardware, software, databases, learning object repositories, mathematical knowledge management, collaborative technology) in academia, education and industry.

- To make University of Newcastle a world-leading institution for Computer Assisted Research Mathematics and its Applications.¹

¹2010 ERA. UofN received only ‘5’ in Applied Maths.
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Communication and Computation: are entangled


CARMA’s Deep History

A co-evolution of symbolic/numeric (hybrid) computation, experimental maths, collaborative technology and HPC.

Experimentally-found modular fractal took 3 hrs to print

1982 PBB & JMB ‘minor’ work on fast computation at Dalhousie; experimental mathematicians before term was current.²

1993-03 Moved to SFU and founded Centre for Experimental and Constructive Mathematics (www.cecm.sfu.ca)

1995 Organic Mathematics Project: www.cecm.sfu.ca/organics

2004-09 JMB opens D-Drive (Dalhousie Distributed Research Institute and Virtual Environment) with Canada Research Chair funding

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Roughly **40** current Members and Associates:

- Steering Committee (Assoc Directors for Applied/Pure/OR)
- External Advisory Committee (IBM, Melbourne, LBNL)
- Members and Students from Newcastle
- Associate Members from Everywhere
- Scientific, Administrative and AGR Officers

Frequent visitors: both student and faculty, short and long-term
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- **Regular Colloquia and Seminar Series**
  - NUOR, SigmaOpt, Discrete Maths, Analysis and Number Theory

  - ANZIAM SIGMAopt AGR Seminar with UoSA and RMIT
  - Trans Pacific Workshop: with UBC-O and SFU (monthly-ish)
  - Short Lecture Series (2-5 lectures)
    - 2010 Rockafellar on *Risk* and Diestel on *Haar measure*
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    - 2013 Ioffe on *Semi-algebraic Opt*, Lasserre on *Moment problems*

- **AMSI Honours (MSc) Courses** (over 400 hours pa)

- **International Workshops and Conferences**: including
  - IP Down Under for INFORS 2011 (July 6-8, 2011)
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Our Services Include

**AGR** Grid-enabled connected-rooms for classes, seminars, meetings:
- **V205** for dis-located collaboration;
- **V206** for co-located collaboration.

**HPC** 110 core MacPro Cluster and x-grid plus access to NSW and National computing services.

**Web Services** include:
- **DocServer** http://docserver.carma.newcastle.edu.au:
  CECM → DDRIVE → CARMA Archie → Mosaic → Google
- **Inverse symbolic calculator (ISC Plus)**
  http://isc.carma.newcastle.edu.au
- **BBP digit database** http://bbp.carma.newcastle.edu.au
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   - Inverse problems & Phase reconstruction
   - Projection methods & Entropy optimization
   - Signal & (Medical) Image reconstruction

2. Nonlinear Functional Analysis
   - Convex analysis & Monotone operators (with Liangjin Yao)
   - Geometric fixed point theory

3. Computational Number Theory
   - Arithmetic of random walks
   - MZVs & Lattice sums; Mahler measures

4. Algorithmic Complexity Theory
   - Fast high precision Special functions
   - Multidimensional quadrature (for fractals)
   - CAS and Maths visualization (and 3D)

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   - MZVs & Lattice sums; Mahler measures

4. **Algorithmic Complexity Theory**
   - Fast high precision Special functions
   - Multidimensional quadrature (for fractals)
   - CAS and Maths visualization (and 3D)
My Current Research Interests include:

1. **Optimization Theory and Applications**
   - Inverse problems & Phase reconstruction
   - Projection methods & Entropy optimization
   - Signal & (Medical) Image reconstruction

2. **Nonlinear Functional Analysis**
   - Convex analysis & Monotone operators (with Liangjin Yao)
   - Geometric fixed point theory

3. **Computational Number Theory**
   - Arithmetic of random walks
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Symbolic-Numeric-Graphic Computation: **SNAG**

Square distance to origin \((11/16)\) and between points \((3/8)\) in fractal carpet

Michael Rose: work motivated by senile rat brains

CARMA and Me, 2012
Symbolic-Numeric-Graphic Computation: SNAG

Square distance to origin (11/16) and between points (3/8) in fractal carpet
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J.M. Borwein
CARMA and Me, 2012
The Fractal Nature of Me:

Examples of each of the 4 items follow

1. Divide and Concur: Douglas-Rachford reconstruction methods (Fran & BS)

2. Optimization Texts: Convex Functions a 2011 Choice Outstanding Academic Title

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To find a point on a sphere and in an affine subspace

Briefly, a visual theorem is the graphical or visual output from a computer program — usually one of a family of such outputs — which the eye organizes into a coherent, identifiable whole and which is able to inspire mathematical questions of a traditional nature or which contributes in some way to our understanding or enrichment of some mathematical or real world situation.

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3. Three Ramblers: A. Straub, J.J. Borwein, J. Wan

2011. AS won ACM-ISSAC Best Student Paper prize
JW was B.H. Neumann prize winner
3. Moments of Random Walks (Flights):

Definition (Moments and Challenging integrals)

For complex \( s \) the \( n \)-th **moment function** is

\[
W_n(s) = \int_{[0,1]^n} \left| \sum_{k=1}^{n} e^{2\pi x_k i} \right|^s dx
\]

\[
= \int_{[0,1]^{n-1}} \left| 1 + \sum_{k=1}^{n-1} e^{2\pi x_k i} \right|^s d(x_1, \ldots, x_{n-1})
\]

Thus, \( W_n := W_n(1) \) is the **expectation**.

- So

\[
W_2 = 4 \int_{0}^{1/4} \cos(\pi x) \, dx = \frac{4}{\pi}
\]

and \( W_2(s) = \left( \frac{s/2}{s} \right) \) (combinatorics).
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For complex $s$ the $n$-th moment function is

$$W_n(s) = \int_{[0,1]^n} \left| \sum_{k=1}^{n} e^{2\pi i k} \right|^s \, dx$$

$$= \int_{[0,1]^{n-1}} \left| 1 + \sum_{k=1}^{n-1} e^{2\pi i k} \right|^s \, dx(1, \ldots, x_{n-1})$$

Thus, $W_n := W_n(1)$ is the expectation.

- So

$$W_2 = 4 \int_0^{1/4} \cos(\pi x) \, dx = \frac{4}{\pi}$$

and $W_2(s) = \binom{s/2}{s}$ (combinatorics).
3. One 1500-step Walk in the plane: a familiar picture

2D and 3D lattice walks are different:

A drunk man will find his way home but a drunk bird may get lost forever.
— Shizuo Kakutani
3. 50, 100, 1000 3-step Walks: a less familiar picture?

\[ W_3(1) = \frac{16 \sqrt[3]{4} \pi^2}{\Gamma(\frac{1}{3})^6} + \frac{3\Gamma(\frac{1}{3})^6}{8 \sqrt[3]{4} \pi^4} \]
3. Moments of a Three Step Walk: in the complex plane

Theorem (Tractable hypergeometric form for $W_3$)

(a) For $s \neq -3, -5, -7, \ldots$, we have

$$W_3(s) = \frac{3^{s+3/2}}{2\pi} \beta \left( s + \frac{1}{2}, s + \frac{1}{2} \right) 3F_2 \left( \frac{s+2}{2}, \frac{s+2}{2}, \frac{s+2}{2} \mid 1, \frac{1}{4} \right).$$

(2)

(b) For every natural number $k = 1, 2, \ldots$,

$$W_3(-2k - 1) = \frac{\sqrt{3} \left( \frac{2k}{k} \right)^2}{2^{4k+1} 3^{2k}} 3F_2 \left( \frac{1}{2}, \frac{1}{2}, \frac{1}{2} \mid k + 1, k + 1 \mid \frac{1}{4} \right).$$
3. Moments of a Four Step Walk

Theorem (Meijer-G form for $W_4$)

For $\Re s > -2$ and $s$ not an odd integer

$$W_4(s) = \frac{2^s \Gamma(1 + \frac{s}{2})}{\pi} \frac{\Gamma(-\frac{s}{2})}{\Gamma(-\frac{s}{2})} G_{22}^{22} \left( \begin{array}{c} 1, \frac{1-s}{2}, 1, 1 \\ \frac{1}{2} - \frac{s}{2}, -\frac{s}{2}, -\frac{s}{2} \end{array} \right| 1 \right). \quad (3)$$

$W_4$ with phase colored continuously (L) and by quadrant (R)
3. Moments of a Four Step Walk

Theorem (Meijer-G form for $W_4$)

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$W_4$ with phase colored continuously (L) and by quadrant (R)
3. Density of a Three and Four Step Walk (BSW, 2010)

\[ p_3(\alpha) = \frac{2}{\pi} \sqrt{3}\alpha \frac{2\sqrt{3}\alpha}{3 + \alpha^2} \binom{2}{2} F_1 \left( \begin{array}{c} 1/3, 2/3 \\ 1 \\ \end{array} \left| \frac{\alpha^2 (9 - \alpha^2)^2}{(3 + \alpha^2)^3} \right. \right) \]

For \( n \geq 7 \) the asymptotics \( p_n(x) \sim \frac{2x}{n} e^{-x^2/n} \) are good. (These are hard to draw.)

\[ p_4(\alpha) = \frac{2}{\pi^2} \sqrt{16 - \alpha^2} \frac{\sqrt{16 - \alpha^2}}{\alpha} \text{Re} \quad 3F_2 \left( \begin{array}{c} 1/2, 1/2, 1/2 \\ 5/6, 7/6 \\ \end{array} \left| \frac{(16 - \alpha^2)^3}{108 \alpha^4} \right. \right) \]
4. Pi Photo-shopped: a 2010 Pi Day Contest

Royal Society: “Nullius in Verba” (trust not in words)

Many mathematicians: “Noli Credere Pictis”
4. Life of Pi

- At the end of his story, Piscine (Pi) Molitor writes

> I am a person who believes in form, in harmony of order. Where we can, we must give things a meaningful shape. For example — I wonder — could you tell my jumbled story in exactly one hundred chapters, not one more, not one less? I'll tell you, that's one thing I hate about my nickname, the way that number runs on forever. It's important in life to conclude things properly. Only then can you let go.

- We may not share the sentiment, but we should celebrate that Pi knows Pi to be irrational.
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We may not share the sentiment, but we should *celebrate* that Pi knows Pi to be irrational.
4. Why Pi?  “Pi is Mount Everest.”

**What motivates modern computations of** $\pi$ **— given that irrationality and transcendence of** $\pi$ **were settled a century ago?**

- One motivation is the raw challenge of harnessing the stupendous power of modern computer systems.

Programming is quite hard — especially on large, distributed memory computer systems: load balancing, communication needs, etc.

**Substantial practical spin-offs accrue:**

- Accelerating computations of $\pi$ sped up the fast Fourier transform (FFT) — heavily used in science and engineering.
- Also to bench-marking and proofing computers, since brittle algorithms make better tests.
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- Beyond practical considerations are fundamental issues such as the normality (digit randomness and distribution) of $\pi$.

  John von Neumann so prompted ENIAC computation of $\pi$ and $e$ — and $e$ showed anomalies.

  - Kanada, e.g., made detailed statistical analysis — without success — hoping some test suggests $\pi$ is not normal.
    - The 10 decimal digits ending in position one trillion are $6680122702$, while the 10 hexadecimal digits ending in position one trillion are $3F89341CD5$.

  - We still know very little about the decimal expansion or continued fraction of $\pi$. We can not prove half of the bits of $\sqrt{2}$ are zero.
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4. Pi seems ‘Random’: Things we sort of know about Pi

Fran Aragon’s 2.873 GB walk on a 200 billion binary digits of Pi

- A Poisson inter-arrival time model applied to 15.925 trillion bits gives: probability Pi is not normal less than one part in is $10^{3600}$.

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At work Haifa, May 2012
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\[\text{Bailey, Borwein, Calude, Dinneen, Dumitrescu, and Yee, “An empirical approach to the normality of pi.”}\]

4. Pi seems Random: Some million step bit walks

Euler’s constant and a pseudo-random number

Fractal dimension of random walks of 1 million steps

Fractal dimension of 10,000 random walks of 1 million steps

Fractal dimension of first 500 thousand step walk

Fractal dimension of whole walk
4. Pi seems Normal: Compare to Stoneham’s number \[ \sum_{k \geq 1} 1/(3^k2^{3^k}), \] 1

- **b-Normal**: all length \( n \) \( b \)-ary strings occur with prob. \( 1/b^n \)
- In base 2 Stoneham’s number is provably normal (Left)
- It may be normal base 3 (Right)
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- And in many other bases. We should have drawn pictures earlier!
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4. Pi seems Random and Normal: Compared to Human Genomes

Genomes are ‘just’ base four numbers.

The X Chromosome (34K) and Chromosome One (10K).
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The X Chromosome (34K) and Chromosome One (10K).
Erdős-Copeland number (concatenated primes, base 2) and Champernowne number (concatenated integers, base 4).

- All pictures thanks to Fran Aragon and Jake Fountain http://www.carma.newcastle.edu.au/jon/numtools.pdf
4. Pi Seems Normal: Comparisons to other provably normal numbers

Erdös-Copeland number (concatenated primes, base 2) and Champernowne number (concatenated integers, base 4).

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4. Pi is Still Mysterious: Things we don’t know about Pi

We do not ‘know’ (in the sense of being able to prove) whether ....

- The simple continued fraction for Pi is unbounded. 
  - Euler found the 292.
- There are infinitely many sevens in the decimal expansion of Pi.
- There are infinitely many ones in the ternary expansion of Pi.
- There are equally many zeroes and ones in the binary expansion of Pi.
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4. Animation, Simulation and Stereo

The latest developments in computer and video technology have provided a multiplicity of computational and symbolic tools that have rejuvenated mathematics and mathematics education. Two important examples of this revitalization are experimental mathematics and visual theorems — ICMI Study 19 (2012)

Cinderella, 3.14 min of Pi, Catalan’s constant and Passive Three D
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Thank You to All

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