SEVEN LECTURES on

Variational Analysis

Generalized Nash Equilibrium Problems, Bilevel programming and MPEC

CIMPA-UNESCO-MESR-MINECO-INDIA
New Delhi, India, November 25-December 6

Laureate Professor
University of Newcastle, NSW

Director, Centre for Computer Assisted Research Mathematics and Applications

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Seven Lectures on VA

Introduction to variational analysis. I will lecture (primarily) out of


Lecture 1 and 2: Introduction to Variational analysis and Variational principles. (Ch 1 and Ch 2)
Lecture 3 and 4: Nonsmooth analysis: normal cones and subdifferentials of lsc functions, Fréchet and limiting calculus, coderivatives and their calculus rules. (Ch 3 and Ch 4)
Lecture 5 and 6: Multifunction analysis: sequences of sets, continuity of maps, minimality. (Ch 4 and Ch 5)
Lecture 7: Applications to distance functions, convex analysis and monotone operators. (Ch 5)
Variational arguments are classical techniques whose use can be traced back to the early development of the calculus of variations and further. Rooted in the physical principle of least action, they have wide applications in diverse fields. This book provides a concise account of the essential tools of infinite-dimensional first-order variational analysis illustrated by applications in many areas of analysis, optimization and approximation, dynamical systems, mathematical economics and elsewhere. The book is aimed at both graduate students in the field of variational analysis and researchers who use variational techniques, or think they might like to. Large numbers of (guided) exercises are provided that either give useful generalizations of the main text or illustrate significant relationships with other results.

Jonathan M. Borwein, FRSC is Canada Research Chair in Collaborative Technology at Dalhousie University. He received his Doctorate from Oxford in 1974 and has been on faculty at Waterloo, Carnegie Mellon and Simon Fraser Universities. He has published extensively in optimization, analysis and computational mathematics and has received various prizes both for research and for exposition.

Qiji Zhu is a Professor in the Department of Mathematics at Western Michigan University. He received his doctorate at Northeastern University in 1992. He has been a Research Associate at University of Montreal, Simon Fraser University and University of Victoria, Canada.
I shall follow Sir Lawrence Bragg (1890-1971)

“I feel so strongly about the wrongness of reading a lecture that my language may seem immoderate.

... The spoken word and the written word are quite different arts.

... I feel that to collect an audience and then read one's material is like inviting a friend to go for a walk and asking him not to mind if you go alongside him in your car.”
Techniques of Variational Analysis

✓ I intend to show you the general patterns, flavours of proofs and some of the applications of modern (convex, smooth and nonsmooth) variational analysis.

✓ You will see bits and pieces in the other lectures (sometimes before and sometimes after I lecture on the topic).

✓ The whole text has been converted to a lecture presentation: download here.

✓ So I expect you to read ahead (or afterwards) and find more details there ....
AN ESSENTIALLY STRICTLY CONVEX FUNCTION WITH
NONCONVEX SUBGRADIENT DOMAIN
AND WHICH IS NOT STRICTLY CONVEX

\[ \max[(x-2)^2+y^2-1,-(x^4+y^4)(1/4)] \]
ABSTRACT

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ABSTRACT

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Lecture 3 and 4: Nonsmooth analysis: normal cones and subdifferentials of lsc functions, Fréchet and limiting calculus, coderivatives and their calculus rules. (Ch 3 and Ch 4) [26th and 27th]

Lecture 5 and 6: Multifunction analysis: sequences of sets, continuity of maps, minimality. (Ch 4 and Ch 5) [28th and 29th]

Lecture 7: Applications to distance functions, convex analysis and monotone operators. (Ch 5) [November 30th]
SEVEN LECTURES ON VA

*Lecture 1 and 2*: Introduction to Variational analysis and Variational principles. *(Ch 1 and Ch 2)*

[Lecture 1. 25/11 10.00-11.00]

[Lecture 2. 26/11 10.00-11.00]
Lecture 3 and 4: Nonsmooth analysis: normal cones and subdifferentials of lsc functions, Fréchet and limiting calculus, coderivatives and their calculus rules. (Ch 3 and Ch 4)

[Lecture 3. 26/11 13.45-14.45]
[Lecture 4. 27/11 11.15-12.15]
Lecture 5 and 6: Multifunction analysis: sequences of sets, continuity of maps, minimality. (Ch 4 and Ch 5)

[Lecture 5. 28/11 9.00-10.00]
[Lecture 6. 29/11 14.45-15.45]
Lecture 7: "Applications" to distance functions, convex analysis and monotone operators. (Ch 5)

[Lecture 7. 30/11 9.00-10.00]
SEVEN LECTURES ON VA

The end

“Regeneration”