A Branch-Price-and-Cut Algorithm for a Maritime Inventory Routing Problem

Faramroze Engineer [1]
Kevin Furman [2]
George Nemhauser [3]
Martin Salvelsbergh [3]
Jin-Hwa Song [2]

[1] University of Newcastle
[2] ExxonMobil Research and Engineering
[3] Georgia Institute of Technology

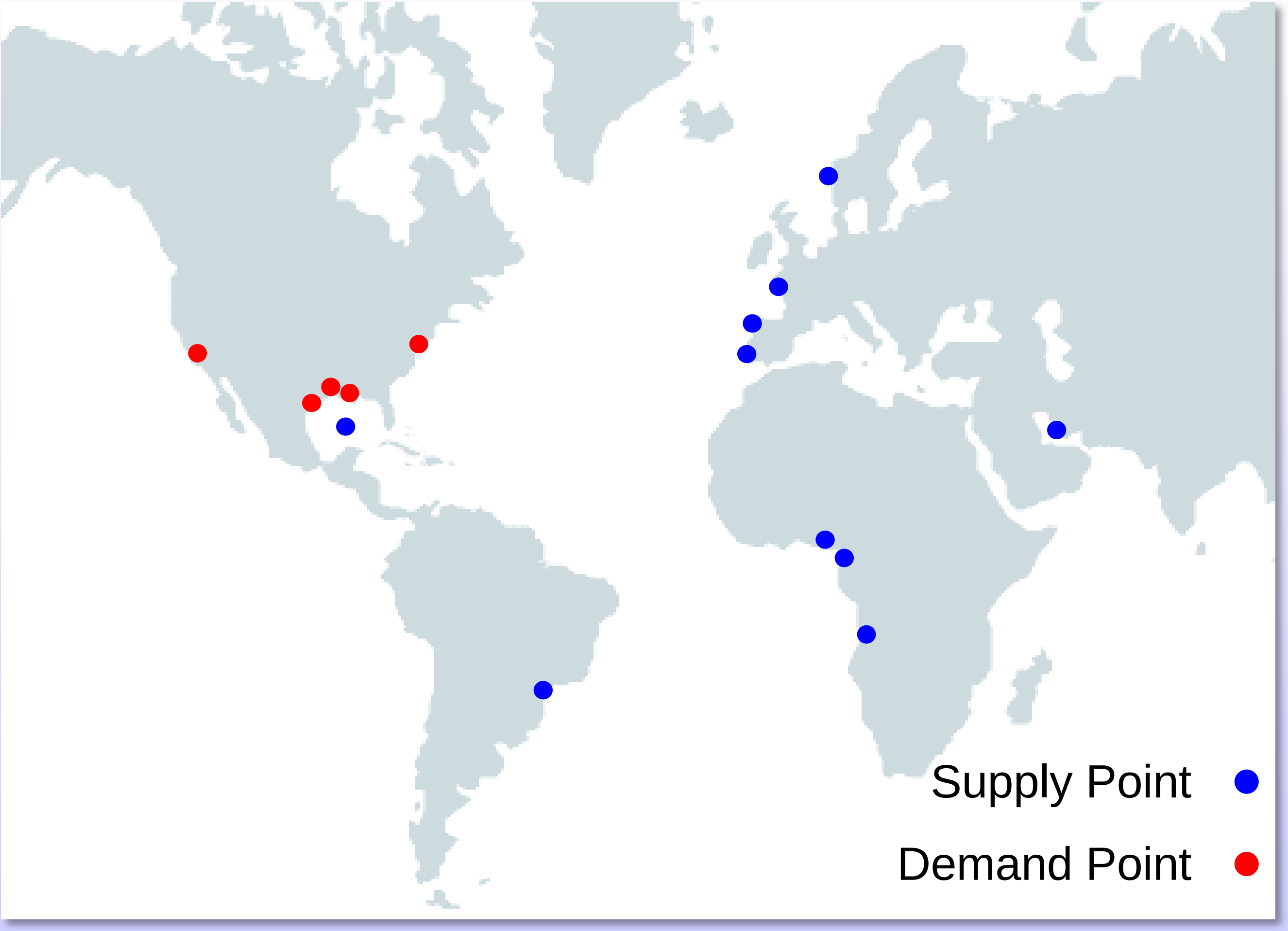
CARMA Workshop

Newcastle, November 1st, 2009

Outline

- IRP in maritime transportation
  - Problem description
- A Time-indexed Column Generation Formulation
- The Pricing Problem
  - problem characteristics
- Cut generation
  - extended VRP cuts
  - new mixed 0-1 cuts
- Branching
- Computational results

Problem Description

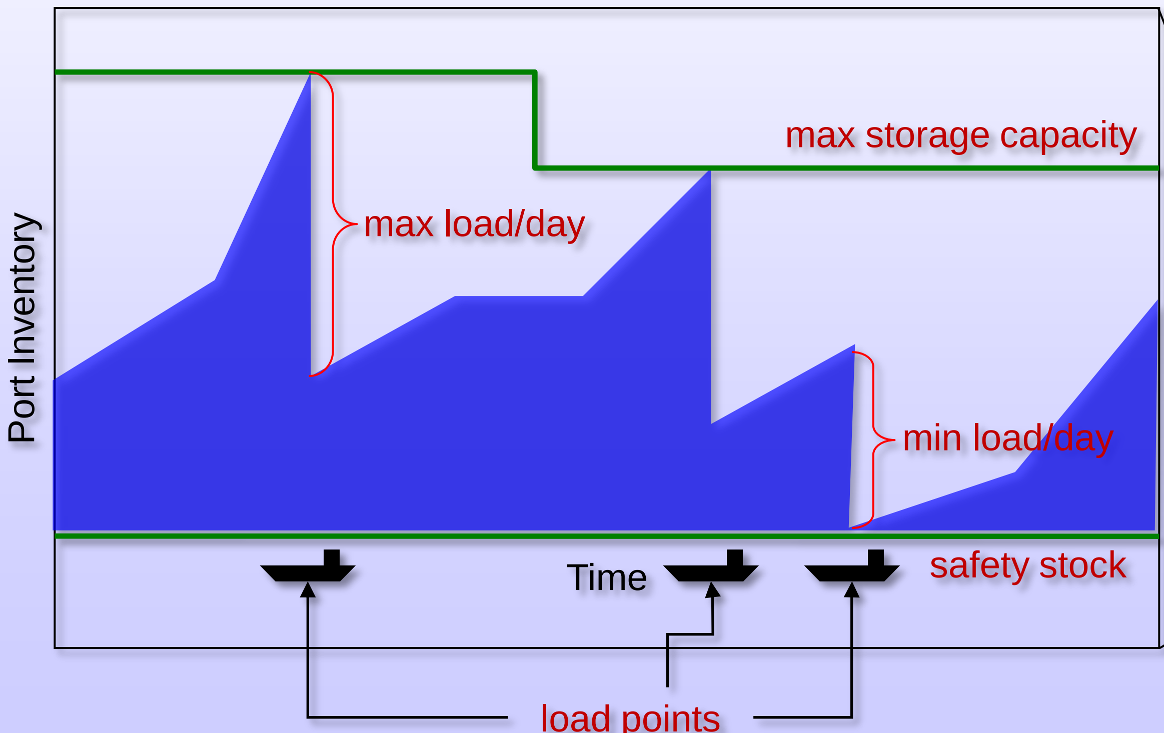


Problem Description (Supply/Load Ports)

Q: Given a production profile for a port, when and how much inventory should be picked up by a vessel?

Supply port constraints:

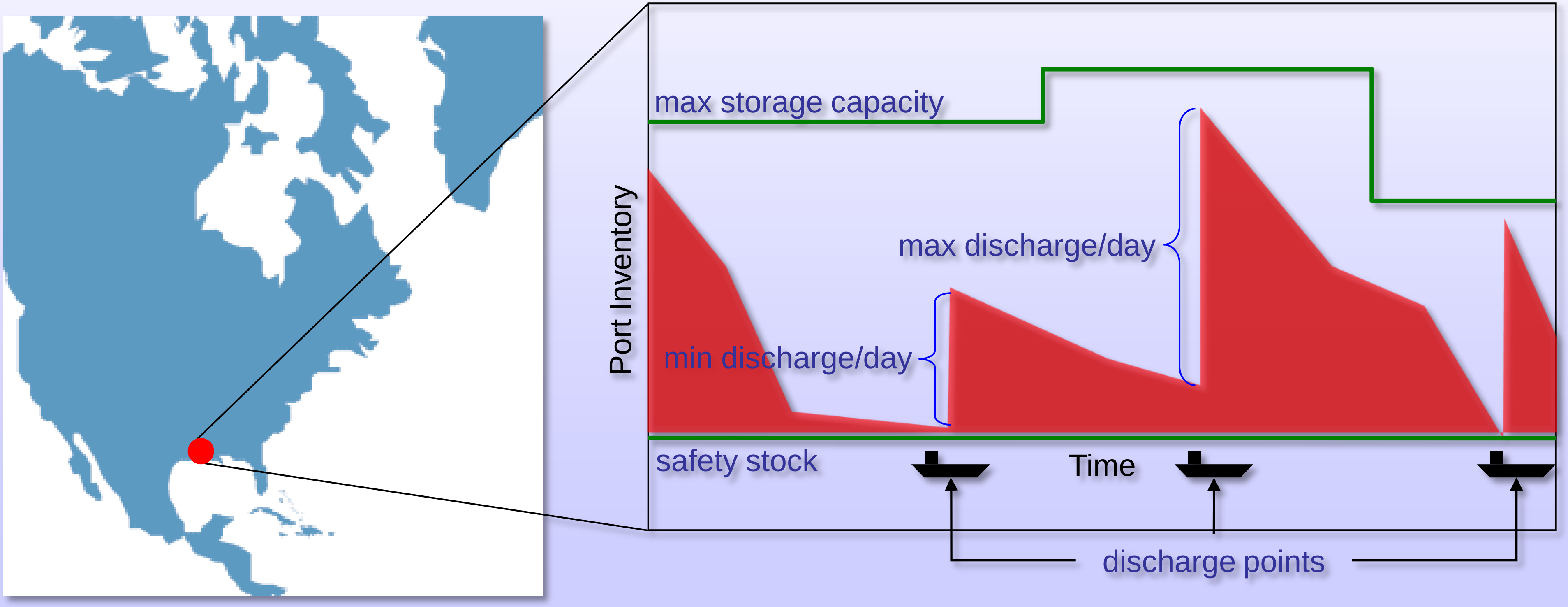
- Port storage capacity and safety stock
- Min and Max load limits per day



Q: Given a demand profile for a supply port, when and how much inventory should be dropped-off by a vessel?

Demand port constraints:

- Port storage capacity and safety stock
- Min and Max discharge limits per day

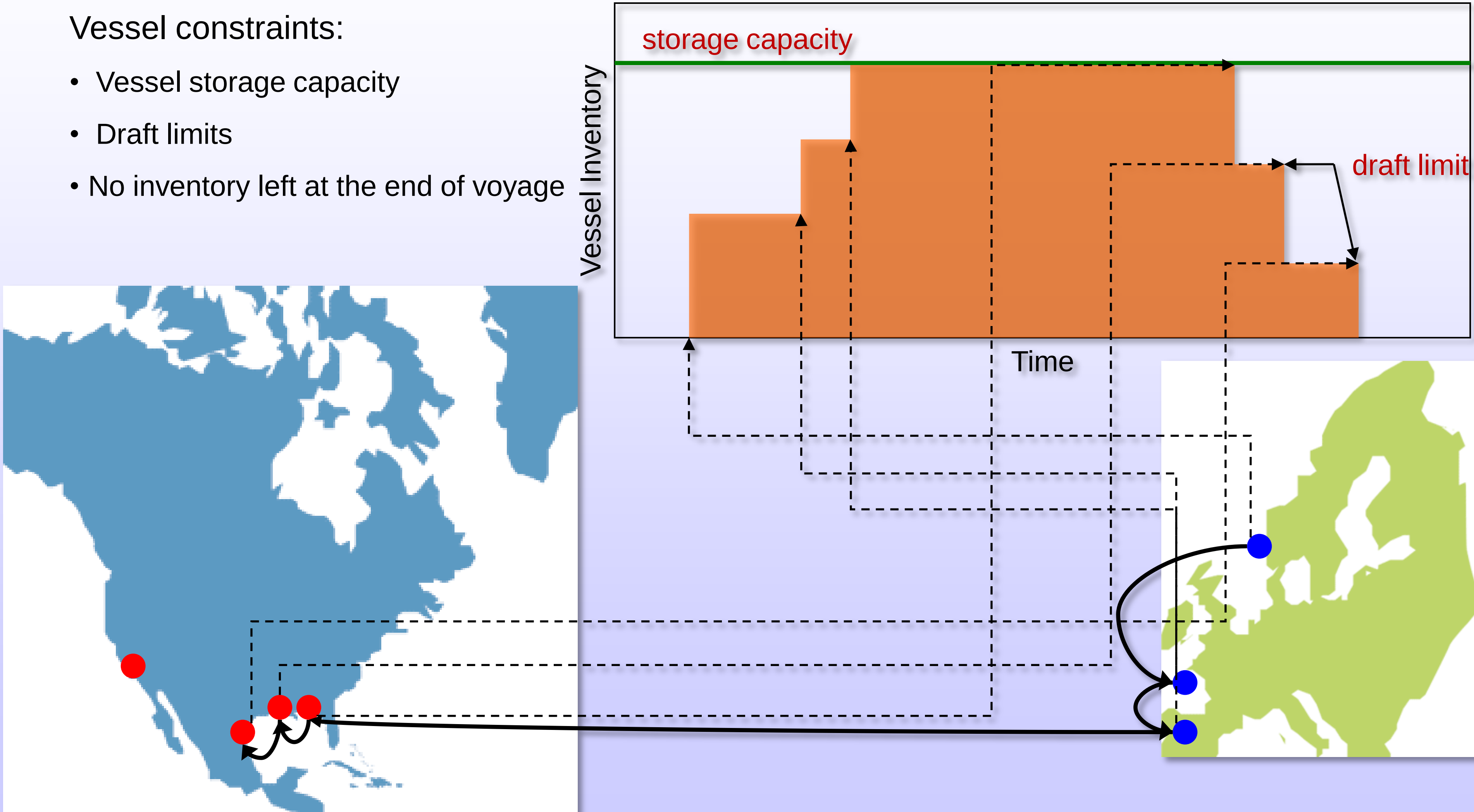


Problem Description (Vessel)

Q: Given a time window of operation, how to route a vessel so that it is available at a port to load/discharge when required?

Vessel constraints:

- Vessel storage capacity
- Draft limits
- No inventory left at the end of voyage



A Time-indexed Column Generation Formulation

The Master Problem

cost of voyage r

amount loaded/discharged at port j and time t

$$\min \sum_{v\in V}\sum_{r\in R_v} c^r \lambda^r$$

$$\text{s.t.} \quad I_{j,t} = I_{j,t-1} + b_{j,t} - \sum_{v\in V}\sum_{r\in R_v} f^r_{j,t}\lambda^r, \qquad j\in J_S,\ t=1,\ldots,T,$$

inventory at port j and time t

$$I_{j,t} = I_{j,t-1} - b_{j,t} - \sum_{v\in V}\sum_{r\in R_v} f^r_{j,t}\lambda^r, \qquad j\in J_D,\ t=1,\ldots,T,$$

production/demand at port j and time t

$$0 \le I_{j,t} \le Q_{j,t}, \qquad j\in J_S\cup J_D,\ t=1,\ldots,T,$$

$$\sum_{r\in R_v}\lambda^r = 1, \qquad v\in V,$$

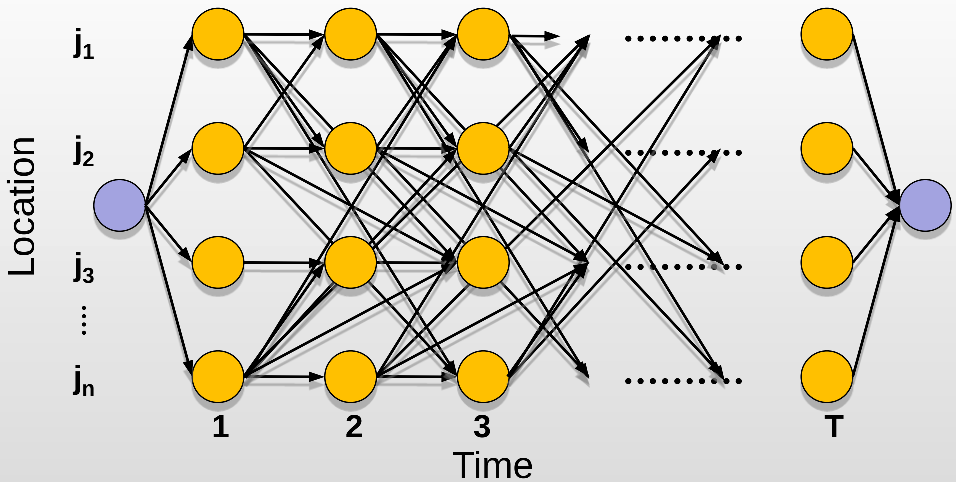
$$\lambda^r \ge 0, \qquad v\in V,\ r\in R_v$$

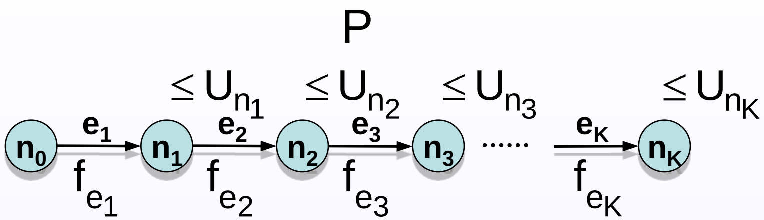
$$\sum_{r\in R_v} z^r_{j,t}\lambda^r \in \{0,1\}, \qquad v\in V,\ j\in J_S\cup J_D,\ t=1,\ldots,T.$$

0-1 indicator of voyage r loading/discharging at port j and time t

The Pricing Problem

Find min cost route and determine quantity loaded/discharged at each port that is visited so that vessel capacity, draft limits, and min/max load/discharge quantities are not exceeded.

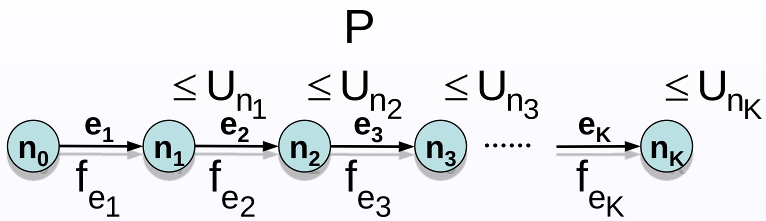




For a given path P, $A(P) = \{e_1,\ldots,e_K\}$ and $N(P) = \{n_0,n_1,\ldots,n_K\}$, an optimal allocation of **load** quantities can be obtained by solving the linear relaxation of the ***Multi-period Knapsack Problem*** (LP-MKP):

$$
\begin{aligned}
\min \quad & \sum_{i=1,\ldots,K} c'_{e_i} f_{e_i} \\
\text{s.t.} \quad & \sum_{i=1,\ldots,j} f_{e_i} \le U_{n_j} \quad \text{for all } j = 1,\ldots,K, \text{ and} \\
& l_{e_i} \le f_{e_i} \le u_{e_i} \quad \text{for all } i = 1,\ldots,K.
\end{aligned}
$$

The Pricing Problem: Characteristics



For a given path P, $A(P) = \{e_1, \ldots, e_K\}$ and $N(P) = \{n_0, n_1, \ldots, n_K\}$, an optimal allocation of **load** quantities can be obtained by solving the linear relaxation of the ***Multi-period Knapsack Problem*** (LP-MKP):

$$\begin{aligned} \min \quad & \sum_{i=1,\ldots,K} c'_{e_i} f_{e_i} \\ \text{s.t.} \quad & \sum_{i=1,\ldots,j} f_{e_i} \le U_{n_j} \text{ for all } j = 1,\ldots,K, \text{ and} \\ & l_{e_i} \le f_{e_i} \le u_{e_i} \text{ for all } i = 1,\ldots,K. \end{aligned}$$

Proposition

LP-MKP can be solved by:

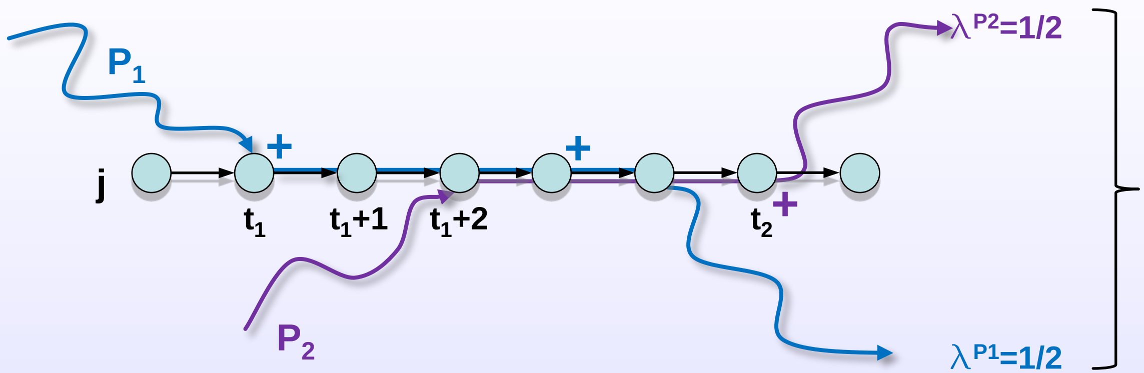
1. Initializing f_{e_i} to l_{e_i} for all $i = 1,\ldots,K$, and
2. increase load quantity on arcs **greedily** (i.e. non-decreasing order of c'_{e_i}) until we either
   - i. reach the upper limit u_{e_i}, or
   - ii. reach some limit U_{n_i} on the total amount of inventory allowed before entering node n_i.

Corollary

There exists an optimal allocation $f^*_{e_i}$ $i = 1,\ldots,K$ such that for each $i = 1,\ldots,K$ either :

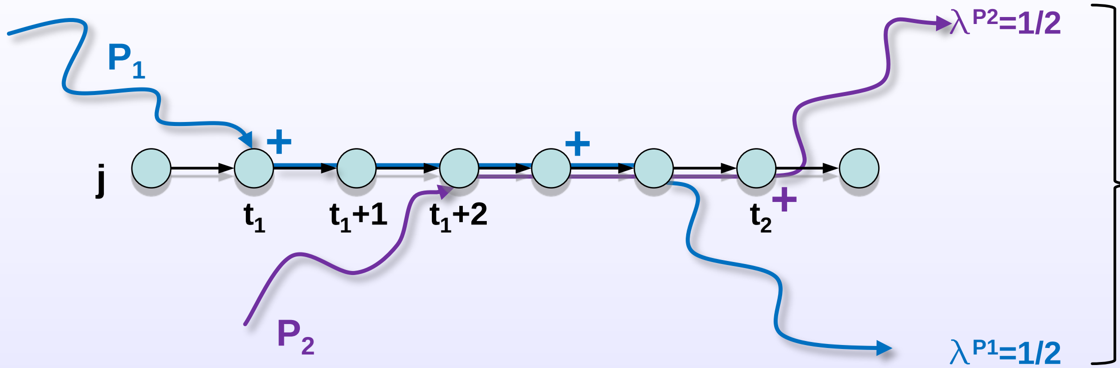
1. $f^*_{e_i} \in \{l_{e_i}, u_{e_i}\}$, or
2. $f^*_{e_i} = U_{n_k} - \sum_{j \in \{1,\ldots,k\} \setminus \{i\}} f^*_{e_j}$ for some $k \ge i$ and $f^*_{e_j} \in \{l_{e_j}, u_{e_j}\}$ for all $j = i+1,\ldots,k$.

Given port j and time interval $[t_1,t_2]$, compute min number of loads/discharges based on excess/deficit inventory and max load/discharge per day.



3/2 loads at j during $[t_1,t_2]$.

Given port j and time interval $[t_1,t_2]$, compute min number of loads/discharges based on excess/deficit inventory and max load/discharge per day.



$$\begin{array}{llll}
\text{Inventory before } t_1 & = I_{j,t_1-1} & = & 50 \\
\text{Production during } [t_1,t_2] & = \sum_{t=t_1,\ldots,t_2} b_{j,t} & = & 25 \times 6 = 150 \\
\text{Capacity at } j \text{ at } t_2 & = Q_{j,t_2} & = & 75 \\
\text{Excess inventory} & = 50+150-75 & = & 125 \\
\text{Max load per day} & = F_j^{max} & = & 75 \\
\text{Min no. of loads at } j \text{ during } [t_1,t_2] & = \left\lceil \frac{125}{75} \right\rceil & = & 2
\end{array}$$

Cuts for IRP: Port capacity cuts

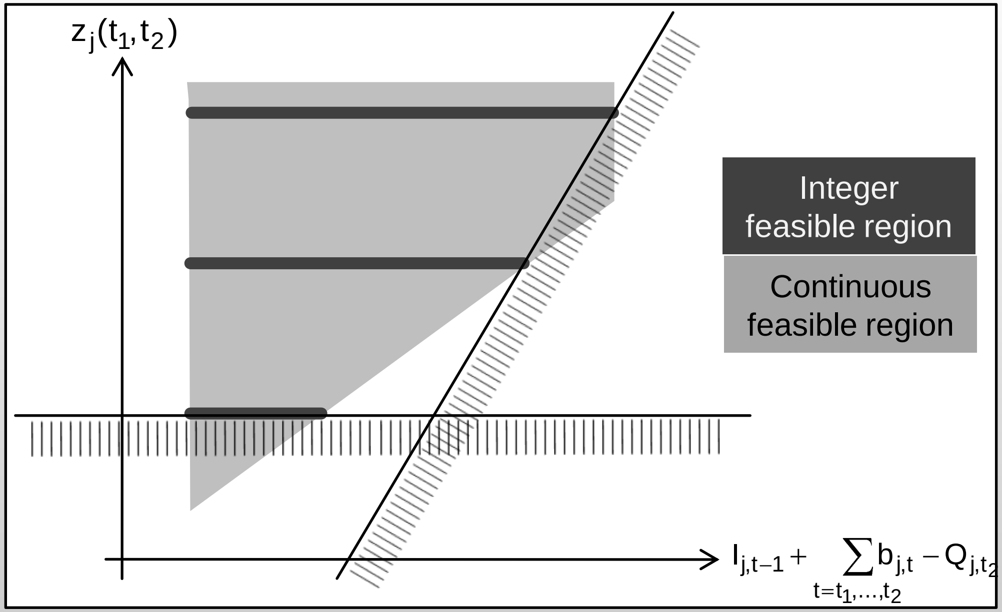
Given port j and time interval $[t_1,t_2]$, compute min number of loads/discharges based on excess/deficit inventory and max load/discharge per day.

$$z_j(t_1,t_2) = \sum_{v \in V} \sum_{r \in R_v} \sum_{t=t_1,\dots,t_2} z^r_{j,t} \lambda^r \geq \left\lceil \frac{I_{j,t_1-1} + \sum_{t=t_1,\dots,t_2} b_{j,t} - Q_{j,t_2}}{F^{max}_j} \right\rceil$$

Given port j and time interval $[t_1,t_2]$, compute min number of loads/discharges based on excess/deficit inventory and max load/discharge per day.

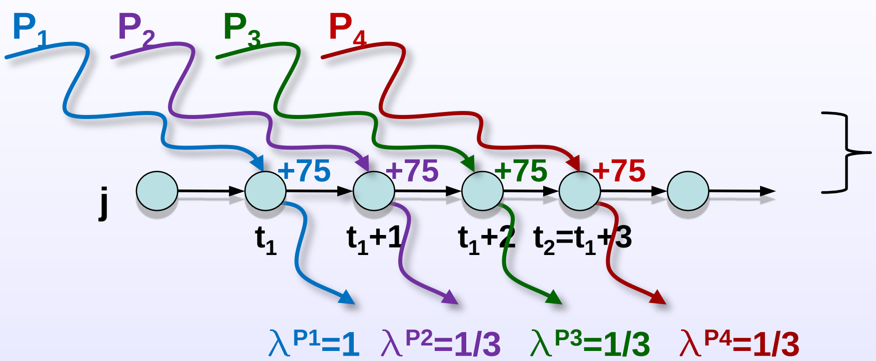
$$z_j(t_1,t_2) = \sum_{v \in V} \sum_{r \in R_v} \sum_{t=t_1,\dots,t_2} z^r_{j,t} \lambda^r \geq \left\lceil \frac{I_{j,t_1-1} + \sum_{t=t_1,\dots,t_2} b_{j,t} - Q_{j,t_2}}{F^{max}_j} \right\rceil$$

$$0 \leq I_{j,t_1-1} \leq Q_{j,t_1-1}$$



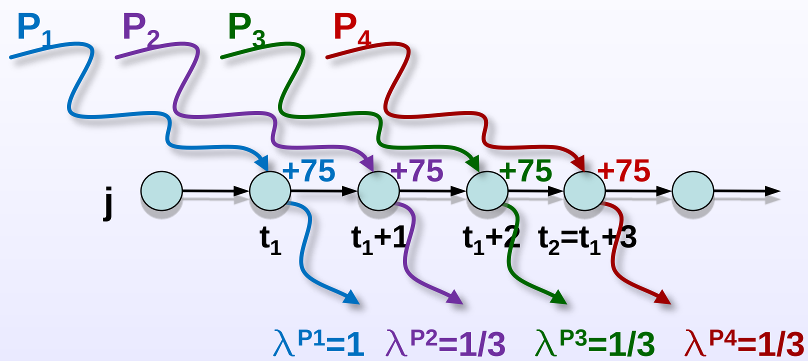
Cuts for IRP: Timing cuts

Given port j and time interval $[t_1,t_2]$, compute timing of departures based on excess/deficit inventory, production/demand rate, and vessel capacity.



- At least 2 loads at j during $[t_1,t_2]$
- At least 2 visits at j during $[t_1,t_2]$
- 0+(1+2+3)/3=2 days "wait" since t_1 to pickup 150 units of inventory during $[t_1,t_2]$.

Given port j and time interval $[t_1,t_2]$, compute timing of departures based on excess/deficit inventory, production/demand rate, and vessel capacity.



- At least 2 loads at j during $[t_1,t_2]$ ✓
- At least 2 visits at j during $[t_1,t_2]$ ✓
- 0+(1+2+3)/3=2 days "wait" since t_1 to pickup 150 units of inventory during $[t_1,t_2]$.

$$
\begin{aligned}
\text{Inventory loaded during } [t_1,t_2] = f_j(t_1,t_2) &= \sum_{v\in V}\sum_{r\in R_v}\sum_{t=t_1,\dots,t_2} f^r_{j,t} &= 150\\
\text{Inventory before } t_1 &= I_{j,t_1-1} &= 50\\
\text{Max load per day} &= F_j &= 75\\
\text{Vessel capacity} &= Q &= 100\\
\text{Production rate during } [t_1,t_2] &= b_{j,t} &= 25
\end{aligned}
$$

$\Rightarrow$ At least 2 visits required and at least one of these must load on or after t_1+3

$\Rightarrow$ To load 150 units of inventory sum of last load time over all vessels $\geq t_1+3$

Cuts for IRP: Timing cuts

Given port j and time interval $[t_1,t_2]$, compute timing of departures based on excess/deficit inventory, production/demand rate, and vessel capacity.

$$\text{If } (k-1)Q < f_j(t_1,t_2) - I_{j,t_1-1} \le kQ \text{ then:}$$

Amount of inventory loaded at j during $[t_1,t_2]$ in excess of what is available at t_1-1

1. at least k visits are required, and
2. at least i visits must load on or after

$$t_1 + \left\lceil \frac{f_j(t_1,t_2) - I_{j,t_1-1} - (i-1)Q}{b_j^{max}(t_1,t_2)} \right\rceil - 1$$

for all $i = 1,\ldots,k$.

Constant overestimation of production rate at j during $[t_1,t_2]$

Given port j and time interval $[t_1,t_2]$, compute timing of departures based on excess/deficit inventory, production/demand rate, and vessel capacity.

$$\text{If } (k-1)Q \le f_j(t_1,t_2) - I_{j,t_1-1} \le kQ \text{ then :}$$

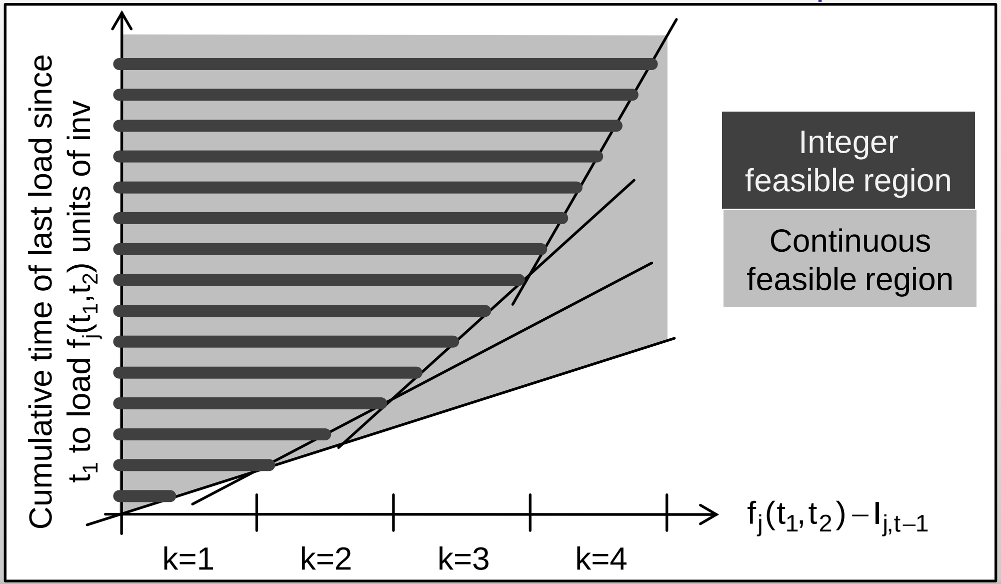
1. at least k visits are required, and
2. at least i visits must load on or after

$$t_1 + \left\lceil \frac{f_j(t_1,t_2) - I_{j,t_1-1} - (i-1)Q}{b_j^{max}(t_1,t_2)} \right\rceil - 1$$

for all $i = 1,\ldots,k$.

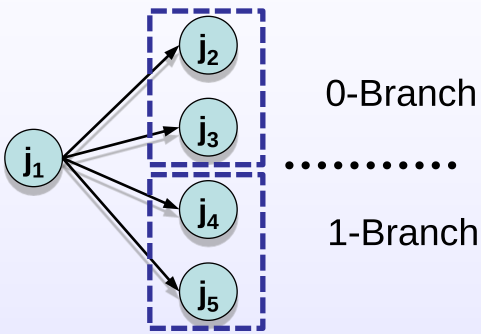
Amount of inventory loaded at j during $[t_1,t_2]$ in excess of what is available at t_1-1

Constant overestimation of production rate at j during $[t_1,t_2]$

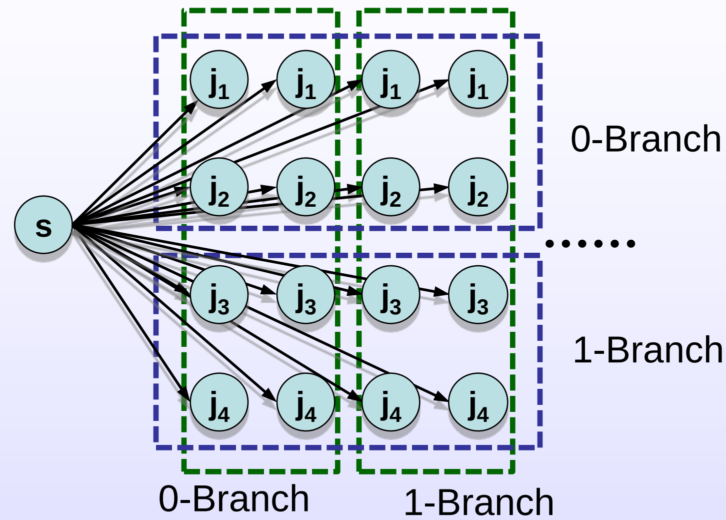


Branching for IRP

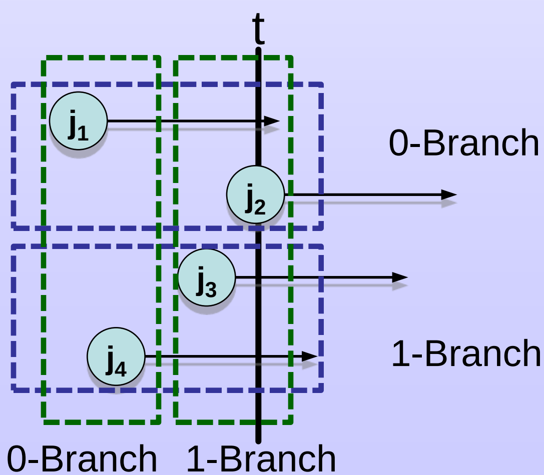
1. Partition follow-on ports



2. Partition location/timing of first/last load/discharge



3. Partition arc cut-set within network



4. Partition decision to load/discharge at port and time

$$\sum_{r \in R_v} z^r_{j,t} \lambda^r \in \{0,1\},$$

Computational Experiments: LP relaxation Results

Inst. Class	No. inst.	Avg. LP gap (%)	Avg. % gap closed after					
			+PP	+BC	+Cuts			
					PCC only	VCC only	TC only	All
(4,2,2,*)	5	45	10	10	23	33	59	65
(5,2,3,*)	5	142	7	7	9	14	65	67
(5,3,2,*)	5	114	2	3	6	6	39	43
(5,3,3,*)	5	24	17	24	29	34	59	62
(6,3,4,*)	5	66	15	17	26	26	61	65
(6,4,3,*)	5	48	14	16	18	20	49	50
(6,4,4,*)	5	27	14	19	30	34	45	49
(6,4,6,*)	5	10	14	17	44	54	32	62
(6,6,4,*)	5	28	11	13	26	32	31	45

PP – Preprocessing, BC – Boundary constraints, PCC – Port capacity cuts, VCC – Vessel capacity cuts, TC – Timing cuts

Computational Experiments: IP Results

Inst. Class	No. inst.	No. solved inst.			Avg. gap (%)			Avg. time (s)		
		B&C	B&C+	BP&C	B&C	B&C+	BP&C	B&C	B&C+	BP&C
(4,2,2,*)	5	5	5	5	0	0	0	7,584	38	11
(5,2,3,*)	5	2	4	5	71	14	0	21,097	9,621	908
(5,3,2,*)	5	2	4	5	58	44	0	29,925	14,476	1,199
(5,3,3,*)	5	0	1	3	28	15	1.3	36,000	30,033	22,092
(6,3,4,*)	5	0	0	1	49	40	11	36,000	36,000	31,844
(6,4,3,*)	5	0	0	0	83	63	12	36,000	36,000	36,000
(6,4,4,*)	5	0	0	0	47	35	9.3	36,000	36,000	36,000
(6,4,6,*)	5	0	0	2	13	8	1.7	36,000	36,000	28,722
(6,6,4,*)	5	0	0	1	37	29	12	36,000	36,000	32,404

B&C – Default CPLEX11.1

B&C+ - CPLEX11.1 + branching and cut enhancements

BP&C – Our branch-price-and-cut algorithm

Note: 10 hour time limit used

Summary

- A time-indexed column generation formulation
  - Demurrage time and costs (i.e. vessel idle and holding costs)
  - Capacities and production/consumption rates fluctuate over time
  - Enforce draught limits and require no inventory on the vessel at the end of its voyage
- A unique mixed 0-1 pricing problem
  - Extract properties amiable for solving exactly and efficiently through DP
- Cuts
  - Extend VRP capacity cuts to mixed 0-1 case
  - Developed new mixed 0-1 cuts specifically for IRP
- Computational results compare very favorably in terms of producing strong lower bounds as compared to an alternative arc-flow formulation and branch-and-cut approach.

Questions?